On Dynamic Adjustment and Comparative Statics Via the Implicit Function Theorem

Anne-Christine Barthel* and Eric Hoffmann*

*West Texas A&M University

INTRODUCTION AND INTUITION

- Comparative statics analysis asks: "How does the equilibrium (the expected outcome) change when an underlying parameter changes?"
- For example: How does the market price charged in oligopoly competition change if a tax is levied on the producers?
- For empiricists, we can write this question as the following regression equation, where the dependent variable a^* denotes the market price and t denotes the tax rate:
  $a^* = \beta t + \varepsilon$
- What sign do we expect on β?
- In Figure 1 below, BR_1 and BR_2 show the optimal prices for firm 1 and firm 2 dependent on the other firm's price for two levels of taxation with $t' > t$. The initial equilibrium at the initial tax t is denoted by a^*, the new equilibrium at the new tax rate t' is denoted by a^{**}. We see that the equilibrium decreases, hence $\beta < 0$.

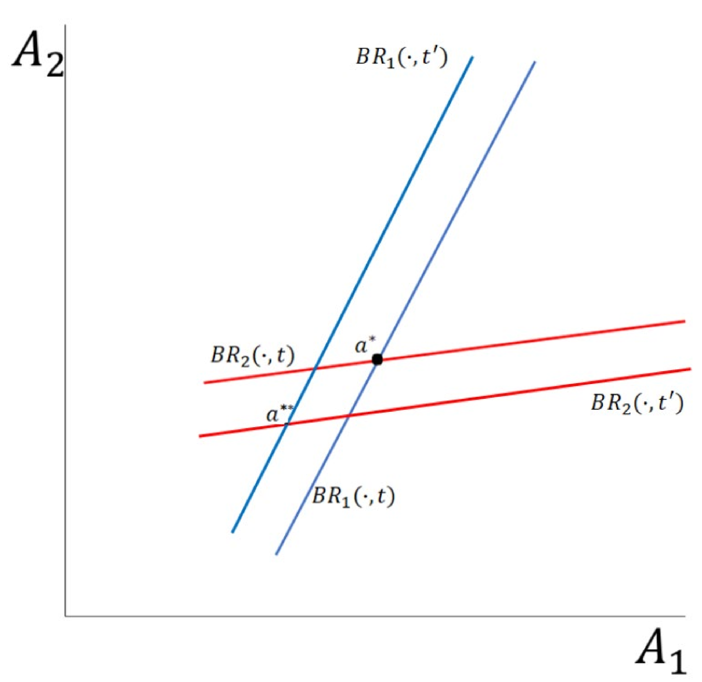

Figure 1

- One key question however remains: In real life, firms don't get automatically placed at the new equilibrium a^{**}, but must learn to adjust from the old equilibrium to the new one.

INTRODUCTION AND INTUITION (CONT.)

- In our example, the learning process suggests that players should come to learn the new equilibrium over time, as Figure 2 illustrates.

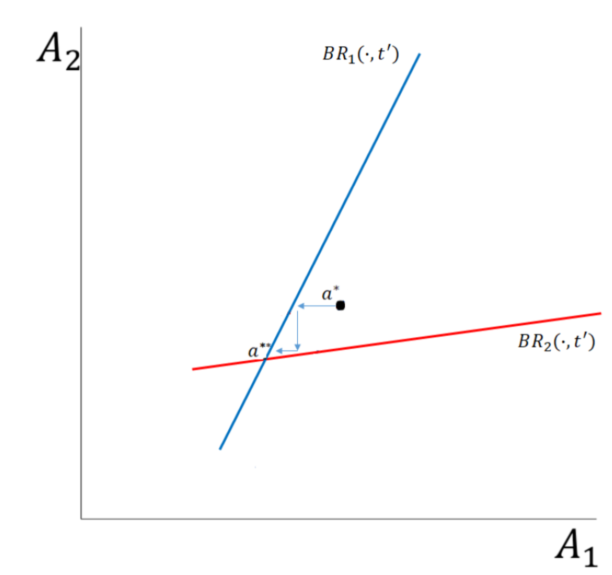

Figure 2

- Unfortunately, this may not always be the case, as the next example in Figure 3 illustrates.
- If we collected data at equilibrium x_2, then our theoretical prediction would suggest that the new equilibrium x_2' would be lower. However, learning dynamics suggest that we should eventually see a higher equilibrium.

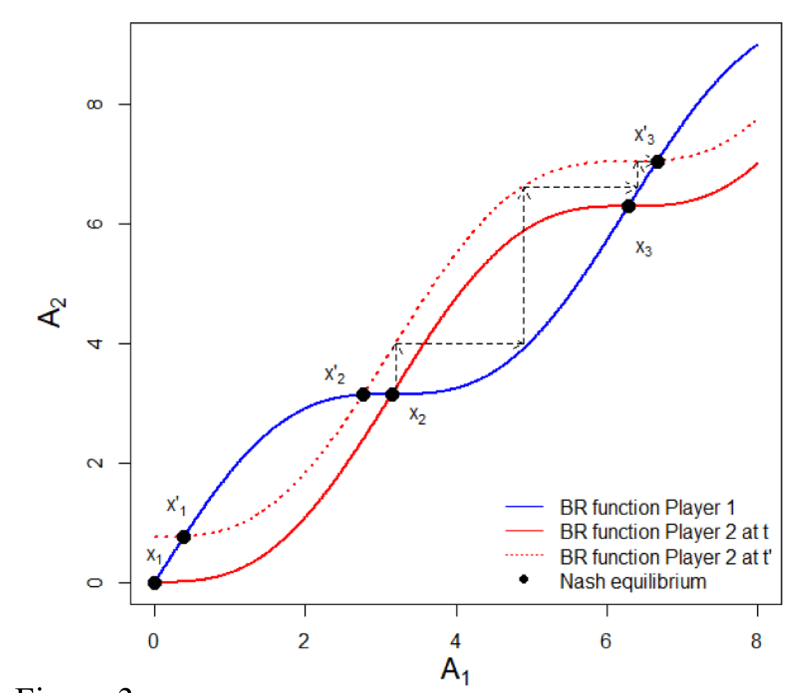

Figure 3

- This contradiction motivates the main question of this paper: Under what conditions can we be confident that our equilibrium prediction after the parameter change will be consistent with the direction in which subsequent learning dynamics will take us? In other words, when will theoretical predictions (which don't incorporate learning) be consistent with regression results?
- Our paper provides a sufficient condition that can be checked within the model which will allow us to be confident that regression results and learning dynamics will all "point in the same direction", or be consistent with one another.

MAIN RESULT

- The intuition for our main result suggests that theoretical predictions and learning dynamics are consistent for stable equilibria (that is, after small deviations away from the equilibrium, learning dynamics lead us back to the same equilibrium).

- **Theorem:**
  Under mild continuity assumptions, if a^* is an equilibrium at an original parameter t, then our theoretical predictions will be consistent with subsequent learning dynamics if a^* is a stable equilibrium.

- Immediate considerations:
  - For empiricists, this is comforting news, as data was likely collected at a stable equilibrium, since unstable equilibria tend to not persist for a long time.
  - For theorists, we would like a simple way to check which equilibria are stable.

- Condition:
  If the spectral radius of the Jacobian matrix

  $$J(a^*) = \begin{pmatrix} 0 & \frac{\partial BR_1}{\partial a_2}(a^*) \\ \frac{\partial BR_2}{\partial a_1}(a^*) & 0 \end{pmatrix}$$

  at an equilibrium a^* is less than 1, or

  $$\rho(J(a^*)) < 1,$$

  then the equilibrium is stable.

- This provides us with a way of checking which equilibria within our model will be consistent with empirical analysis.