On the Existence of Stable Equilibria in Monotone Games

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INTRODUCTION

- In strategic situations, we say we are at an **equilibrium** of the game if no individual has an incentive to change their current strategy.
- An equilibrium is said to be **locally stable** if players will return to that equilibrium after a small deviation in strategy.
- **Games of Strategic Complements (GSC)**: Games with complementary effects, or an incentive to choose a more aggressive strategy if opponents do so as well. For example, price wars.
- It is well-established in the literature of GSC that if these games have a unique equilibrium, it is globally stable (that is play will return for any deviation in strategy, not just small ones).
- If a game has multiple equilibria, neither one can be globally stable.

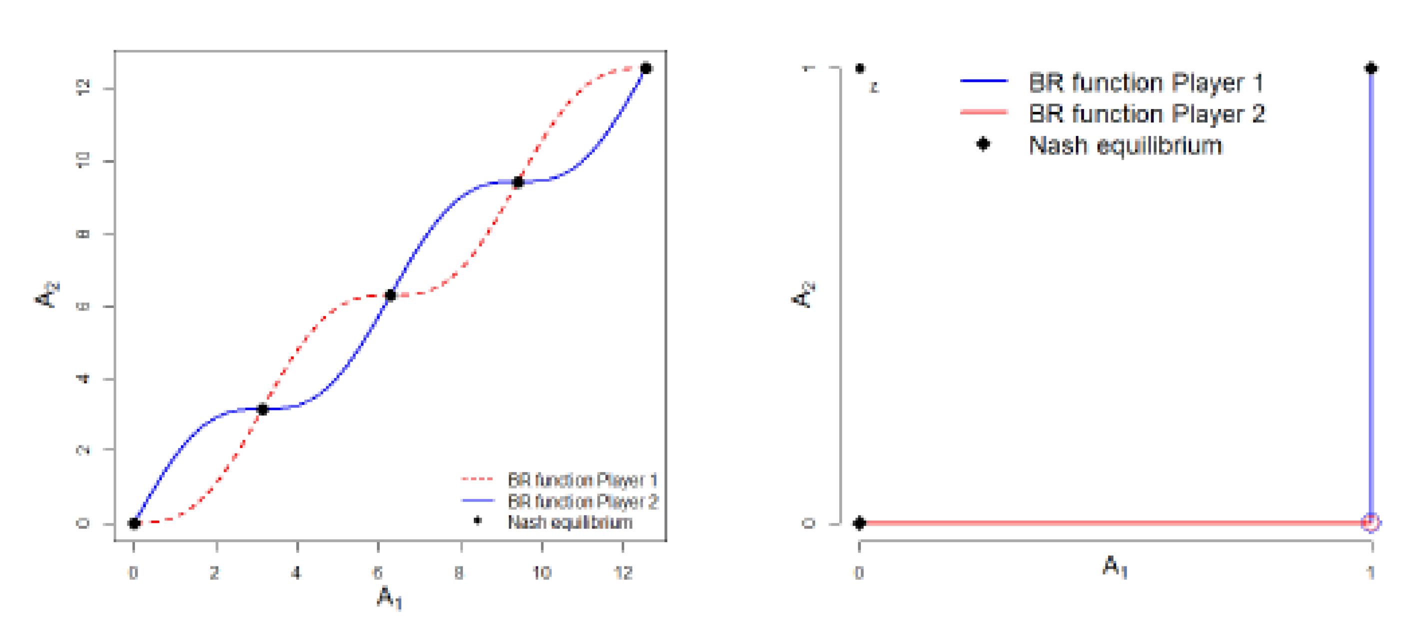
CONTRIBUTION

- This paper asks the following question: Under what conditions can we guarantee a locally stable equilibrium to exist in a GSC?
- This paper provides conditions under which at least one locally stable equilibrium exists in such games. Unlike other results in the literature, our conditions do not rely on differentiability of the underlying payoff functions.
- Moreover, we show that if an equilibrium is locally stable, then the next smallest and next largest equilibrium cannot be locally stable. This implies that a game with two equilibria has exactly one locally stable one.

MOTIVATION

- Although results concerning the existence and stability of extremal equilibria have been established, these cannot be used to guarantee the general existence of a stable equilibrium.
  - Milgrom and Roberts (1990) establishes that the smallest and largest equilibrium are monotonically increasing in a parameterized GSC.
  - Echenique (2002) establishes that monotone equilibria are locally stable.
  - One may hope that through clever parameterization one can guarantee that the extremal equilibria are necessarily stable.
  - The examples below however show that this need not be the case.
- Figure 1(a) illustrates an example of a GSC in which the largest and smallest equilibrium are not locally stable. To see this, suppose players deviate a little to the southwest of the largest equilibrium. Then both players' best responses will be to further decrease their strategies, moving away from the largest equilibrium. Similarly, they will move upwards and away from the smallest equilibrium.
- Figure 1(b) shows an example in which neither one of the two equilibria in a GSC is locally stable. Starting from any point in the interior, player 1 best responds by playing action 1, while player 2 will best respond by playing 0. Then, player 1 will want to play 0, while player 2 will want to best respond with action 1 and play will oscillate between (0,1) and (1,0). Hence neither one of the equilibria is locally stable.

Figure 1: Instability in Monotone Games



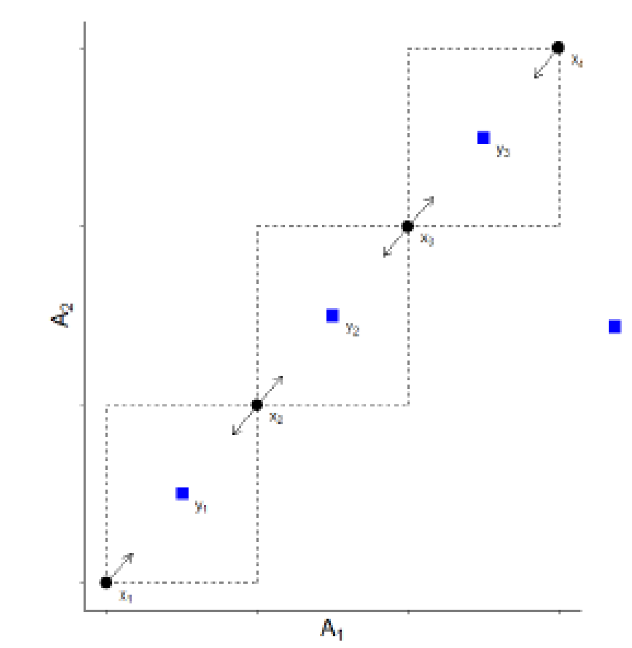
(a) Unstable extremal equilibria

(b) Instability with discontinuous best response

INTUITION OF MAIN RESULT

- Figure 2 illustrates the intuition of our main result.
- y_1, y_2 and y_3 are the three equilibria of a game.
- Notice that the best responses to the largest and smallest points in the joint strategy space are pointing downwards and upwards, respectively.
- The majority of the paper goes towards establishing the existence of points x between any two equilibria such that best responses are either moving upwards or downwards.
- Our strategy is to guarantee that some equilibrium lies in the interior of some $[x_k, x_{k+1}]$ such that best responding to x_k moves upward and best responding to x_{k+1} moves downward.
- For example, if best responding to x_3 moves upward, since best responding to x_4 moves downward, y_3 is locally stable.
- If best responding to x_3 however moves downward and best responding to x_2 moves upward, then y_2 is locally stable.
- Finally, if best responding to x_2 moves downward, then y_1 is locally stable, as best responding to x_1 moves upward.

Figure 2: Intuition of Main Result



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Milgrom, P. & Roberts, J. (1990) Rationalizability, learning and equilibrium in games with strategic complementarities. *Econometrica*, 58(6), 1255-1277.