

# Characterizing Monotone Games

Anne-Christine Barthel\* and Eric Hoffmann\*

\*West Texas A&M University

## INTRODUCTION

- **Games of Strategic Complements (GSC):**  
Games with complementary effects, or an incentive to choose a more aggressive strategy if opponents do so as well. For example, price wars.
- **Games of Strategic Substitutes (GSS):**  
Games with substitute effects, or an incentive to choose a less aggressive strategy if opponents act more aggressively. For example, scaling back output to stabilize the price if competitors increase production.
- **Games of Strategic Heterogeneity (GSH):**  
Games where some players have complementary effects with all players, while others have substitute effects with all players.
- It has been shown that the solution sets of these monotone games (the outcome that we'd expect if players try to maximize their payoffs) possess nice properties.
  - Stability of this outcome
  - Upper and lower bounds on the actions that players will take

## CONTRIBUTION

- This paper asks the following question:  
When can a game that is NOT a GSH (or GSC or GSS) be transformed into one in such a way that the original game can be guaranteed to possess these nice solution properties?
- This paper gives necessary and sufficient conditions for when such a transformation is possible for a very general class of games.
- Surprisingly, we also find that in many settings, “flipping” the orders on some players’ strategy spaces is the only way to transform the game into a GSH.

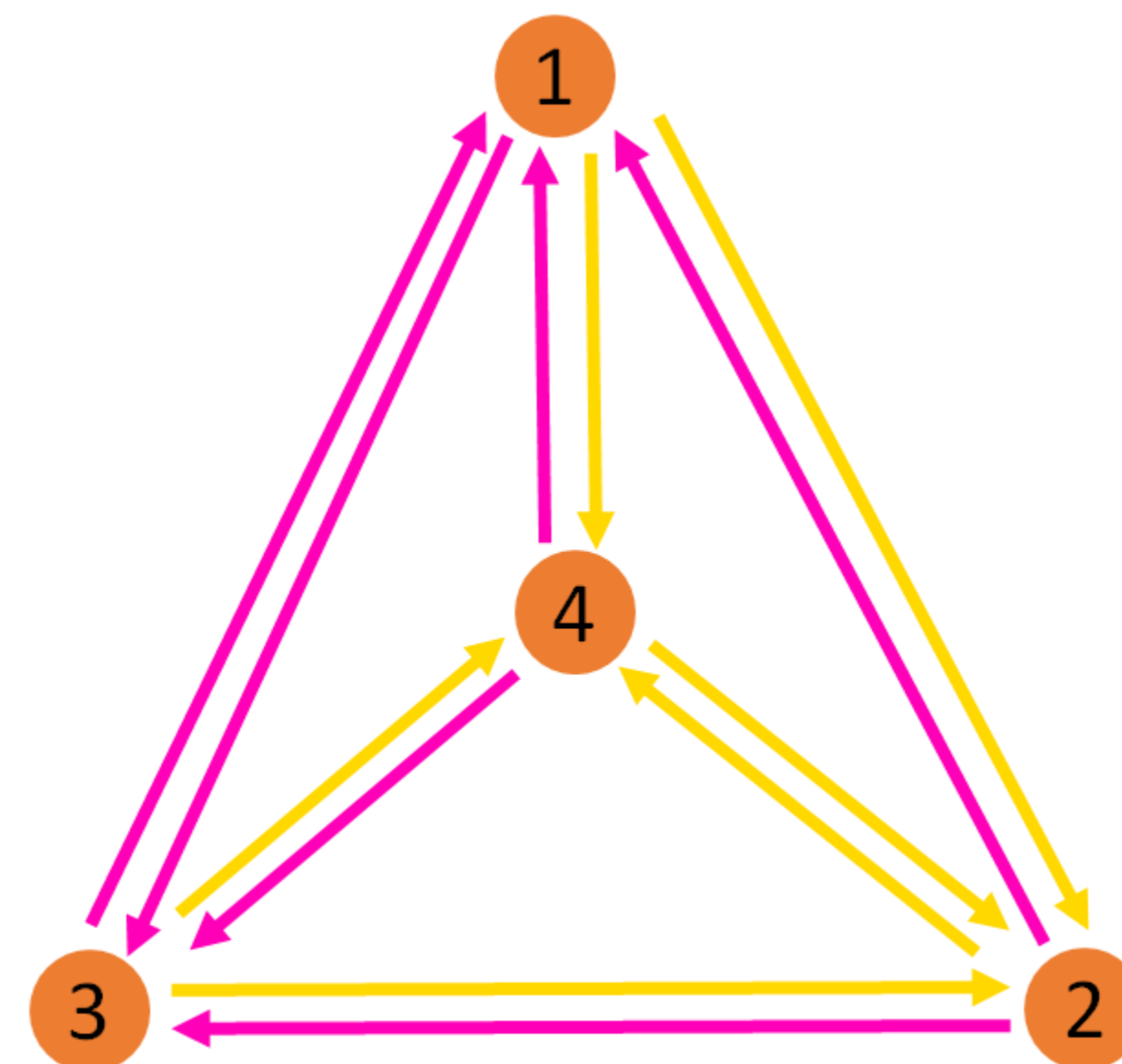
## MODEL AND INTUITION

- We introduce **Games of Mixed Heterogeneity (GMH)**.
- The difference between a GMH and a GSH is that in the former, each player has “pairwise monotonicity”, so that each player’s optimal response to another player’s action can be increasing in the actions of some players (pairwise strategic complements), while decreasing in the actions of others (pairwise strategic substitutes).

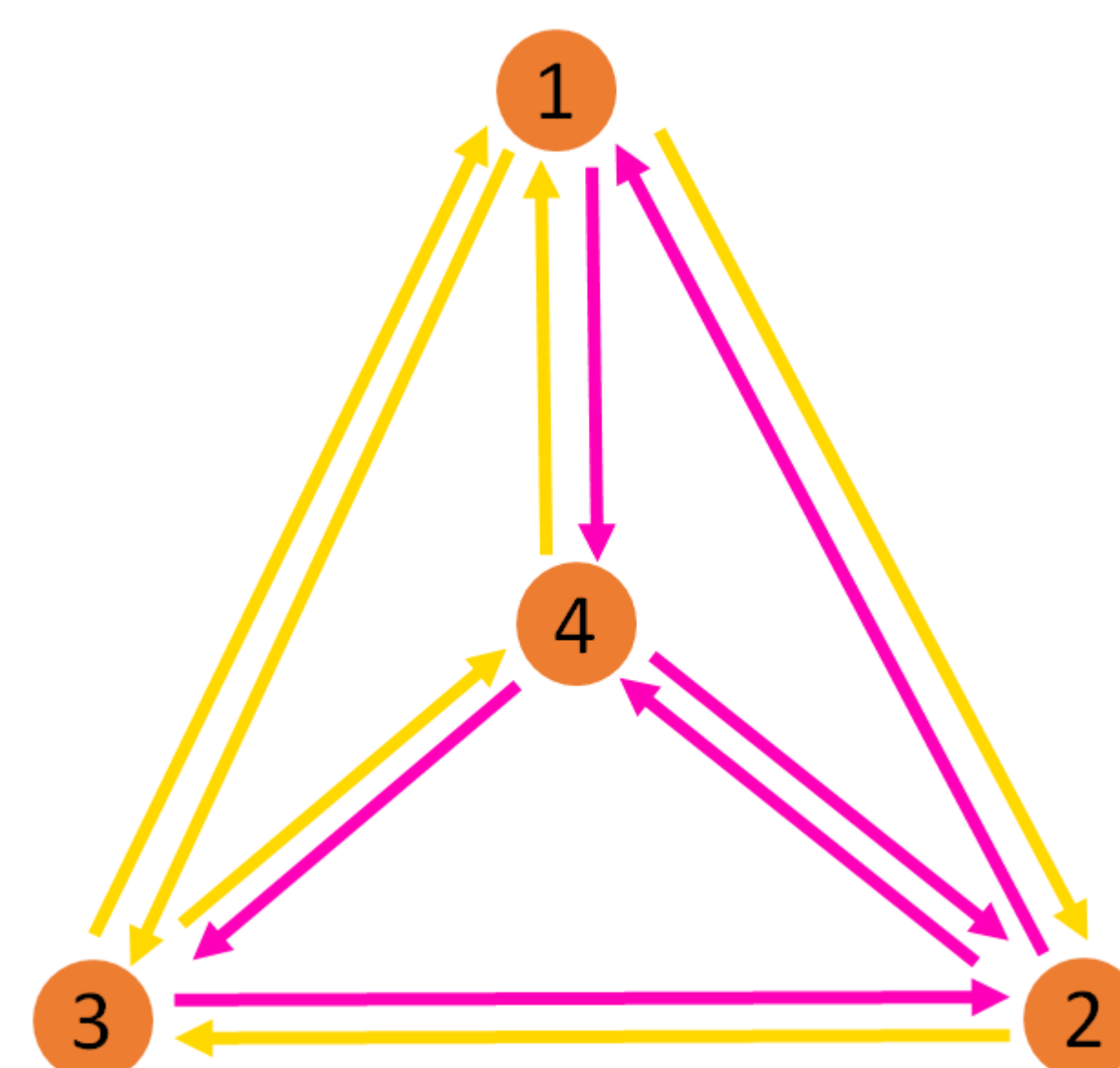
$\pi_1$  exhibits pairwise strategic Substitutes in  $(a_1, a_2)$      $\pi_2$  exhibits pairwise strategic Complements in  $(a_2, a_1)$   
(S for SCHOOL BUS YELLOW)    (C for CANDY PINK)



- In a GSH, all arrows pointing towards each player are of the same color, that is a player has either strategic substitutes or complements with all other players.

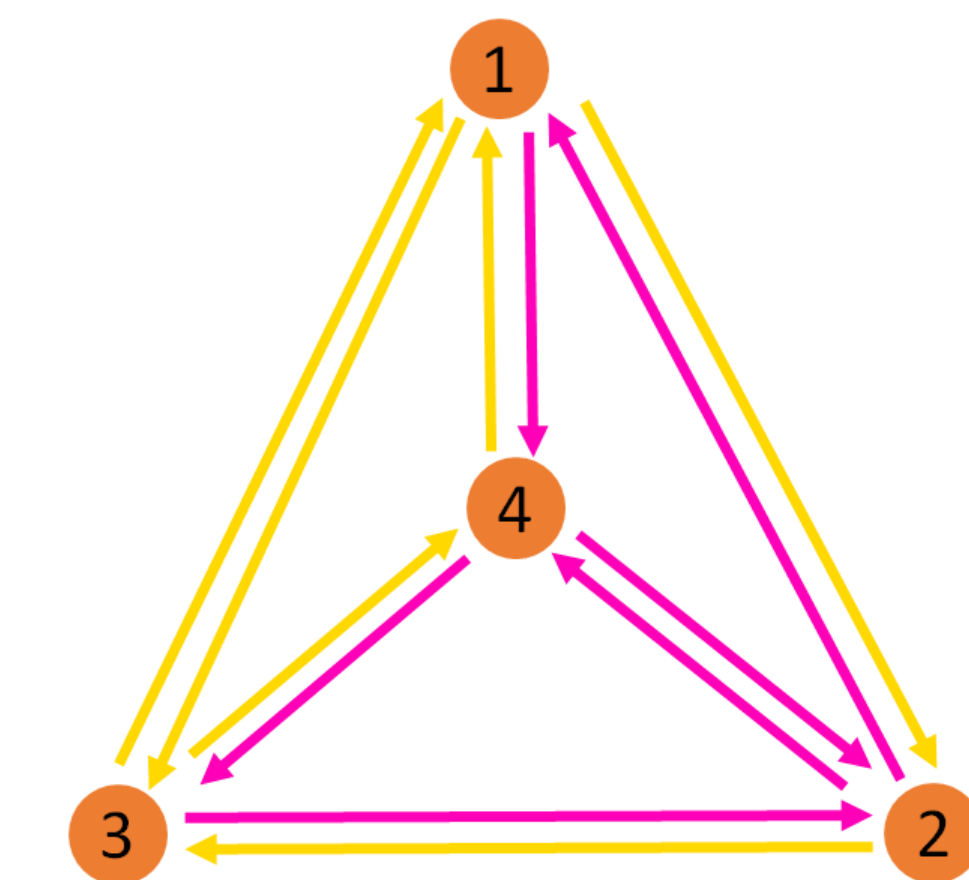


- In a GMH, we can have any combination of arrows between players.

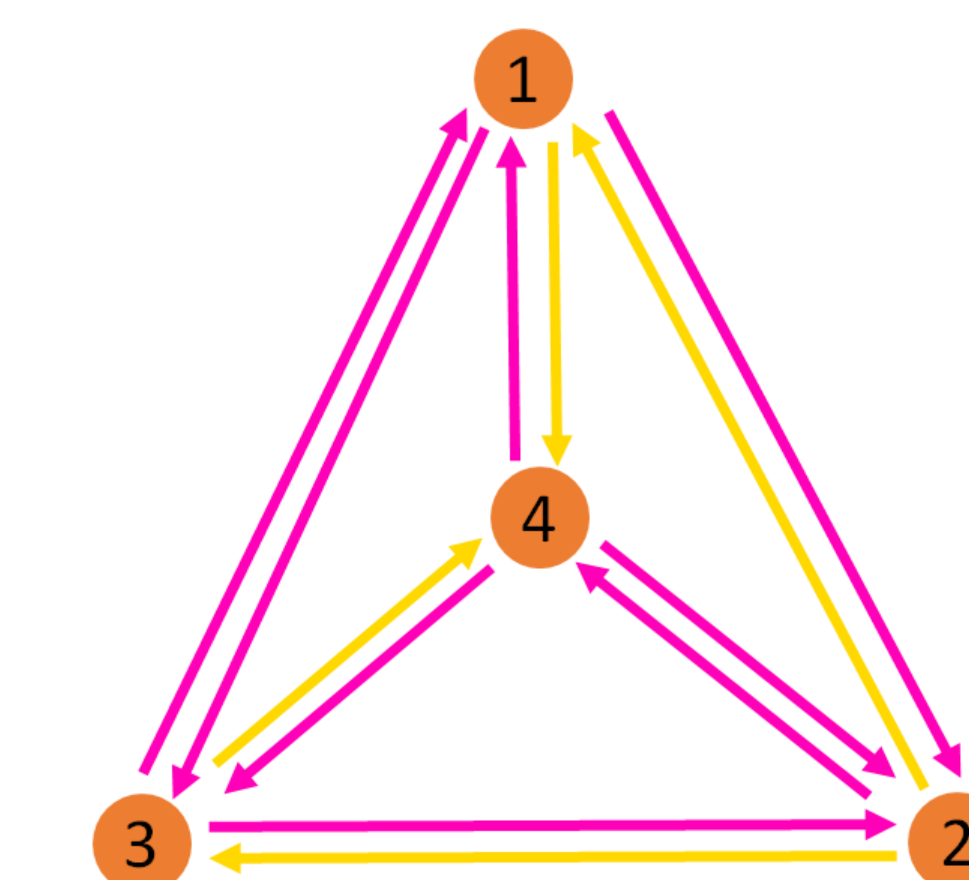


## MAIN RESULT

- A game can be transformed into a GSH by flipping some players’ strategy spaces if and only if we can partition the players into two groups, where each player is affected the same way by all other group members and the opposite way by all non-group members.
- Consider the following initial GMH, where players 1 and 2 are one group and players 3 and 4 are the other group.



- Flipping Player 1’s strategy space will change the color of all of her arrows.



- After also flipping Player 2’s strategy space, we see that the game is now a GSH.

